Exam Symmetry in Physics

Date	January 27, 2010
Room	X 5118.-156
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- You are not allowed to use the lecture notes, nor other notes or books
- All subquestions (a, b, etc) of the three exercises have equal weight
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Exercise 1

Consider the cyclic group C_4: $\text{gp}\{c\}$ with $c^4 = e$.

(a) Determine the order of the elements of C_4.

(b) Write down all proper invariant subgroups of C_4.

(c) To which group is the factor group C_4/C_2 isomorphic?

(d) Construct the character table of C_4.

(e) Construct the three-dimensional vector representation D^V of C_4 and demonstrate whether it is an irrep or not.

(f) Decompose D^V into irreps of C_4 and use the result to conclude whether a crystal with C_4 symmetry can support a permanent electric dipole moment.

(g) Determine the Clebsch-Gordan series of the direct product representation $D^V \otimes D^V$ of C_4.

Exercise 2

Consider the group of rotations in two dimensions $SO(2)$ and the group of unitary 1×1 matrices $U(1)$.

(a) Show that $SO(2) \cong U(1)$.

(b) Give a two-dimensional parametrization of the elements in the Lie algebra of $SO(2)$ and show that upon exponentiation it yields the defining representation of $SO(2)$.

(c) Write down all (complex) irreducible representations of $SO(2)$.

(d) Give an example of a physical system with an $SO(2)$ or $U(1)$ symmetry.

Next consider the extension of $SO(2)$ to include reflections: the group $O(2)$ of orthogonal 2×2 matrices.

(e) Write down the two-dimensional representation of $O(2)$ obtained by its action on the vector

$$\begin{pmatrix} x + iy \\ x - iy \end{pmatrix}$$

(f) Is this two-dimensional rep of $O(2)$ an irrep?

Exercise 3

Consider the group $SU(2)$ of unitary 2×2 matrices with determinant 1.

(a) Give an explicit representation of its generators J_i.

(b) What are the structure constants of $SU(2)$?

(c) What is the Casimir operator of $SU(2)$ and is it an element of the Lie algebra?

(d) Explain why the eigenvalues of the Casimir operator determine the irreps of $SU(2)$.

(e) Give an example of an application of the group $SU(2)$ in physics.